Math 9 — 5.5 Dividing a Polynomial by a Constant — Lesson 6

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4. **a)** $20 \div 5 = 4$

b) $3x \div 3 = x$

c) $(2x + 4) \div 2 = x + 2$

d) $(9x + 6) \div 3 = 3x + 2$

6. **c)** The display shows eight *t*-tiles and twelve –1-tiles arranged as 4 equal rows of two *t*-tiles and three –1-tiles. So, this models $\frac{8t-12}{4}$, which is the quotient in part c

8. **a)** **i)** $\frac{12k}{4} = \frac{12}{4} \times k$

$= 3k$

ii) $(-12k) \div 4 = \frac{(-12)}{4} \times k$

$= (-3) \times k$

$= -3k$

iii) $\frac{12k}{-4} = \frac{12}{(-4)} \times k$

$= (-3) \times k$

$= -3k$

iv) $(-12k) \div (-4) = \frac{(-12)}{(-4)} \times k$

$= 3 \times k$

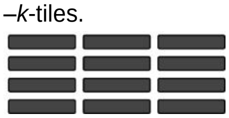
$= 3k$

b) All of the quotients are either $3k$ or $-3k$. The quotient is negative when the dividend and the divisor have different signs and positive when the dividend and the divisor have the same signs.

c) I can use algebra tiles for quotients i and ii in part a.

i) To model $\frac{12k}{4}$ I display twelve *k*-tiles arranged in 4 equal rows. In each row, there are three *k*-tiles.

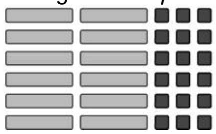
ii) To model $(-12k) \div 4$, I display twelve –*k*-tiles arranged in 4 equal rows. In each row, there are three –*k*-tiles.



The divisions in parts iii and iv cannot be easily modelled with algebra tiles. In part iii, we cannot divide twelve k-tiles into a negative number of equal groups. Instead, I could multiply the numerator and the denominator by (–1) to get an equivalent fraction: $\frac{-12k}{4}$. This can be modelled with twelve –*k*-tiles arranged in 4 equal groups of three –*k*-tiles.

In part iv, I could divide the divisor and the dividend by (–1) to get the equivalent division expression $12k \div 4$. This can be modelled with twelve k-tiles arranged in 4 equal groups of three k-tiles

13. a) Arrange twelve p-tiles and eighteen –1-tiles in 6 equal rows.



In each row, there are two p-tiles and three –1-tiles.

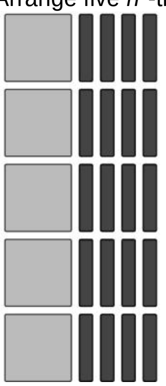
So, $\frac{12p - 18}{6} = 2p - 3$

b) Arrange six $-q^2$-tiles and ten –1-tiles in 2 equal rows.

In each row, there are three $-q^2$-tiles and five –1-tiles.

So, $\frac{-6q^2 - 10}{2} = -3q^2 - 5$

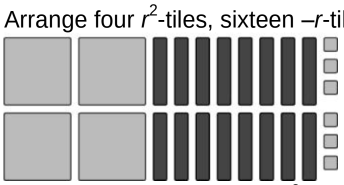
c) Arrange five h^2-tiles and twenty $-h$-tiles in 5 equal rows.



In each row, there is one h^2-tile and four $-h$-tiles.

So, $\frac{5h^2 - 20h}{5} = h^2 - 4h$

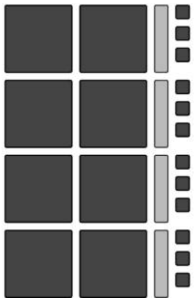
d) Arrange four r^2-tiles, sixteen $-r$-tiles, and six 1-tiles in 2 equal rows.



In each row, there are two r^2-tiles, eight $-r$-tiles, and three 1-tiles.

So, $\frac{4r^2 - 16r + 6}{2} = 2r^2 - 8r + 3$

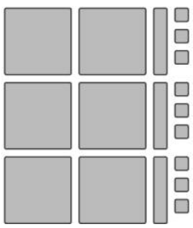
e) Arrange eight $-a^2$-tiles, four a-tiles, and twelve –1-tiles in 4 equal rows.



In each row, there are two $-a^2$-tiles, one a-tile and three -1-tiles.

So, $\dfrac{-8a^2+4a-12}{4} = -2a^2 + a - 3$

f) Arrange six x^2-tiles, three x-tiles, and nine 1-tiles in 3 equal rows.



There are two x^2-tiles, one x-tile, and three 1-tiles in each row.

So, $\dfrac{6x^2+3x+9}{3} = 2x^2 + x + 3$

14. The negative sign of the divisor should apply to all denominators; it is missing in the second and third fractions.
So, $-28m$ divided by -7 gives $+4m$, not $-4m$. Also, 7 divided by -7 simplifies to -1, not 0.
$2m^2 - 4m$ cannot be simplified to $-2m$ because $2m^2$ and $4m$ are unlike terms.
Correct solution:

$(-14m^2 - 28m + 7) \div (-7)$

$= \dfrac{-14m^2}{-7} + \dfrac{-28m}{-7} + \dfrac{7}{-7}$

$= 2m^2 + 4m - 1$

16. a) $\dfrac{24d^2-12}{12}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{24d^2}{12} + \dfrac{-12}{12}$ Simplify each fraction.

$= 2d^2 - 1$

b) $\dfrac{8x+4}{4}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{8x}{4} + \dfrac{4}{4}$ Simplify each fraction.

$= 2x + 1$

c) $\dfrac{-10+4m^2}{-2}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{-10}{-2} + \dfrac{4m^2}{-2}$ Simplify each fraction.

$= 5 - 2m^2$

d) $(25 - 5n) \div (-5)$ Write the quotient expression as a fraction.

$= \dfrac{25-5n}{5}$ Write the quotient expression as the sum of 2 fractions.

$= \dfrac{25}{-5} + \dfrac{-5n}{-5}$ Simplify each fraction.

$= -5 + n$

e) $(-14k^2 + 28k - 49) \div 7$ Write the quotient expression as a fraction.

$= \dfrac{-14k^2+28k-49}{7}$ Write the quotient expression as the sum of 3 fractions.

$= \dfrac{-14k^2}{7} + \dfrac{28k}{7} + \dfrac{-49}{7}$ Simplify each fraction.

$= -2k^2 + 4k - 7$

f) $\dfrac{30-36d^2+18d}{-6}$ Write the quotient expression as the sum of 3 fractions.

$= \dfrac{30}{-6} + \dfrac{-36d^2}{-6} + \dfrac{18d}{-6}$ Simplify each fraction.

$= -5 + 6d^2 - 3d$

g) $\dfrac{-26c^2+39c-13}{-13}$ Write the quotient expression as the sum of 3 fractions.

$= \dfrac{-26c^2}{-13} + \dfrac{39c}{-13} + \dfrac{-13}{-13}$ Simplify each fraction.

$= 2c^2 - 3c + 1$

18. a) i) $(3p)(4) = 12p$

ii) $\dfrac{-21x}{3} = -7x$

iii) $(3m^2 - 7)(-4) = (3m^2)(-4) + (-7)(-4)$
$= -12m^2 + 28$

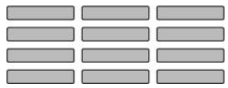
iv) $\dfrac{-2f^2+14f-8}{2} = \dfrac{-2f^2}{2} + \dfrac{14f}{2} + \dfrac{-8}{2}$
$= -f^2 + 7f - 4$

v) $(6y^2 - 36y) \div (-6) = \dfrac{6y^2}{-6} + \dfrac{-36y}{-6}$
$= -y^2 + 6y$

vi) $(-8n + 2 - 3n^2)(3) = (-8n)(3) + 2(3) - 3n^2(3)$
$= -24n + 6 - 9n^2$

b) The products and quotients in parts i, ii, iii, iv, and vi can be modelled with algebra tiles.
The quotient in part v cannot be modelled because we cannot use the algebra tile model to show division by a negative number.

c) i) To model $(3p)(4)$, display three p-tiles repeated 4 times:



ii) To model $\frac{-21x}{3}$, display twenty-one $-x$-tiles arranged in 3 equal rows.

20. a) I know that an equilateral triangle has 3 equal sides. So, I divide the perimeter by 3 to determine the length of one side of the triangle.

$$\frac{15a^2+21a+6}{3} = \frac{15a^2}{3} + \frac{21a}{3} + \frac{6}{3}$$
$$= 5a^2 + 7a + 2$$

The polynomial the represents the length of one side is $5a^2 + 7a + 2$.

b) Substitute $a = 4$ in $5a^2 + 7a + 2$.
$5(4)^2 + 7(4) + 2 = 5(16) + 28 + 2$
$= 110$
The length of one side is 110 cm

23. a) $(3n^2 - 12mn + 6m^2) \div 3 = \frac{3n^2}{3} + \frac{-12mn}{3} + \frac{6m^2}{3}$
$$= n^2 - 4mn + 2m^2$$

b) $\frac{-6rs-16r-4s}{-2} = \frac{-6rs}{-2} + \frac{-16r}{-2} + \frac{-4s}{-2}$
$$= 3rs + 8r + 2s$$

c) $\frac{10gh-30g^2-15h}{5} = \frac{10gh}{5} + \frac{-30g^2}{5} + \frac{-15h}{5}$
$$= 2gh - 6g^2 - 3h$$

d) $(12t^2 - 24ut - 48t) \div (-6) = \frac{12t^2}{-6} + \frac{-24ut}{-6} + \frac{-48t}{-6}$
$$= -2t^2 + 4ut + 8t$$

24. The area of a circle is given by the formula πr^2, where r is the radius.
For the large circle, substitute $r = 3x$ in the formula.
Area of large circle: $\pi(3x)^2 = \pi(3x)(3x)$
$= \pi(3)(3)(x)(x)$
$= 9\pi x^2$

For the small circle, substitute $r = x$.
Area of small circle: $\pi(x)^2 = \pi x^2$

To determine the shaded area in the diagram, subtract the area of the small circle from the area of the large circle.
$9\pi x^2 - \pi x^2 = 8\pi x^2$

A polynomial for the shaded area in the diagram is $8\pi x^2$.